

Problem Set 4  
Choice under Uncertainty

1. Assume an individual with preferences  $A \succ B \succ C \succ D$ . This individual is indifferent between  $B$  and the lottery  $(A, D; 0.4, 0.6)$ . Also, (s)he is indifferent between  $C$  and the lottery  $(B, D; 0.2, 0.8)$ . Construct a set of vN-M utility numbers for the four situations.
2. If we observe that two households spend the same amount on fire insurance for their home, can we conclude that they have the same degree of absolute risk aversion?
3. Suppose a consumer has utility function  $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$ .
  - a) Without taking any monotonic transformations of the original utility function, find the indirect utility function. (Also, explain why a monotonic transformation of the utility function would be undesirable).
  - b) Assume that the consumer's indirect utility function you found in (a) is his/her vN-M utility function. Show that the consumer is risk averse over income shocks.
4. Suppose a lottery that gives 1€ or 25€ with equal probabilities.
  - a) What is the minimum amount you could give someone with  $u(x) = x^{1/2}$  (and who conforms to the vN-M axioms) for not taking this gamble? (Assume this individual has no initial wealth). What is this individual's risk premium? Calculate the probability premium for the individual of (a) and the above lottery.
5. Assume a risk-averse individual who wants to send a package worth  $a$  monetary units. If this person uses the national postal service, there is a chance  $\pi$  that the package will be lost or destroyed in transit. An insurance company offers full insurance against this eventuality for a premium of  $y$ . Calculate  $y$  if the insurance is actuarially fair and show that the individual will prefer to insure.
6. Assume an individual  $i$  with no initial wealth and with utility function  $u(\cdot)$ . This person is indifferent between receiving 90€ with certainty and participating in a lottery that pays 200€ with probability 0.4 and 50€ with probability 0.6. If  $i$  conforms to the vN-M axioms, write down the condition that reflects  $i$ 's indifference, and propose appropriate utility values for  $u(200)$ ,  $u(90)$  and  $u(50)$ . Can you infer the sign of  $u''$ ? What is  $i$ 's risk premium for choosing the lottery?
7. Suppose a consumer  $i$  with initial wealth  $w$ , who conforms to the vN-M axioms. The consumer runs a risk  $p$  of losing  $x$ . If loss of  $x$  occurs, then there is another risk  $q$  that  $i$  loses  $z$  too. An insurance company offers two different programs: in program A,  $i$  can pay a premium  $y$  and get covered for any loss. In program B,  $i$  only pays a fraction  $r < 1$  of the premium, and (s)he is covered for loss of  $x$  only. Write down the conditions that must hold if  $i$  prefers non-insurance to program A, and program A to program B. Then, write down the condition that must hold for program B to be actuarially fair.
8. Suppose two persons  $i$  and  $k$  with utility functions  $u_i(w) = w^{1/2}$  and  $u_k(w) = \ln(w)$ .
  - a) Calculate these persons' Arrow-Pratt measure of risk aversion and show that both  $i$  and  $k$  are risk averse. What condition must hold for  $i$  to be more risk averse than  $k$ ?
  - b) Suppose that individual  $i$ 's wealth is  $w_i$  and that this person is asked to participate in a lottery (for free) that will either multiply  $i$ 's wealth with 4 or diminish it to  $1/4$  of its original level with probabilities 20% and 80% respectively. Show that  $i$  will choose to not take this gamble and calculate the probability premium.
9. For each one of the following statements, argue whether they are true, false or uncertain. Justify your answers.
  - a) Two consumers with identical utility functions have the same absolute risk aversion, and hence, their behaviour in risky situations should be identical.
  - b) vN-M utility functions allow only affine (i.e. positive linear) transformations.
  - c) Regret theory is incompatible with expected utility theory.
  - d) The insurance premium depends on the chosen representation for the utility function.
  - e) If the vN-M utility function is a degree  $n$  polynomial with  $n \geq 2$ , then absolute risk aversion is zero when some individual is extraordinarily wealthy.
10. Mas-Colell, Whinston and Green: Exercises 6.B.4, 6.B.7, 6.C.1, 6.C.15, 6.C.16, 6.C.18