UADPhilEcon
Advanced Microeconomics – Module 1
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Problem Set 1
Preference and Choice

1. Consider a preference relation $\succsim$ on $X=\{x,y,z\}$, such that $x \succsim x$, $x \succsim y$, $x \succsim z$, $y \succsim z$ and $z \succsim x$. Show that $\succsim$ is neither complete nor transitive.

2. Assume that an individual strictly prefers x to y, $x,y \in X=R$, if $x \geq y$.
a) When does this individual find x at least as good as y?
b) Are this individual's preferences rational?

3. Show that $\succ$ is asymmetric (if $x \succ y$, then not $y \succ x$) and negative transitive (for all $x,y,z \in X$, if not $x \succ y$ and not $y \succ z$, then not $x \succ z$), if and only if $\succsim$ is rational.

4. Suppose that someone chooses {cake} when faced with the budget set {bread, cake}, and {bread, butter} when faced with the budget set {bread, butter, cake}.
a) Show that these choices are not consistent with rational behaviour.
b) Is, in your opinion, this person's behaviour reasonable? If yes, how could it conform to the rationality restrictions of economic theory?

5. Consider the following properties for a choice structure $(\mathcal{B},C(\cdot))$:
Property (i): If $x \in B' \subseteq B$, $B \in \mathcal{B}$, $B' \in \mathcal{B}$ and $x \in C(B)$, then $x \in C(B')$.
Property (ii): If $x \in C(B)$, $y \in C(B)$, $B \in \mathcal{B}$ and $y \in C(B')$, $B' \in \mathcal{B}$ and $B \subseteq B'$, then $x \in C(B')$.
a) Show that if a choice structure is consistent with the weak axiom of revealed preference, then it is also consistent with property (i) and property (ii).
b) Show that property (i) is not equivalent to the weak axiom of revealed preference by giving an example of a choice structure that satisfies property (i), but not the weak axiom.
c) Show that property (ii) is not equivalent to the weak axiom of revealed preference by giving an example of a choice structure that satisfies property (ii), but not the weak axiom.
d) Show that a choice structure $(B,C(\cdot))$ has both properties (i) and (ii) if and only if it satisfies the weak axiom of revealed preference.

6. A choice structure $(\mathcal{B},C(\cdot))$ satisfies "condition α" if for any two sets $B \in \mathcal{B}$, $B' \in \mathcal{B}$, if $B \subseteq B'$ and $C(B') \subseteq B$, then $C(B)=C(B')$. Prove that if a choice structure satisfies the weak axiom of revealed preference, then it also satisfies "condition α".

7. Suppose $X=\{x,y,z,w\}$ and $\mathcal{B}=\{\{x,y,z\},\{x,y,z,w\}\}$. List all the sets that $C(\{x,y,z,w\})$ may be equal to, so as to not violate the weak axiom of revealed preference, in the cases where a) $C(\{x,y,z\})=\{x\}$, and b) $C(\{x,y,z\})=\{x,y\}$. For each one of these possible scenarios (in both cases (a) and (b)), is there a unique rational preference relation $\succsim$ to rationalise $C(\cdot)$ relative to $\mathcal{B}$?

8. For a choice structure $(\mathcal{B},C(\cdot))$, suppose that $B \in \mathcal{B}$, $x,y \in B$, $x \in \mathrm{C}(B)$, $B' \in \mathcal{B}$, $x,y \in B'$ and $y \in \mathrm{C}(B')$. Define the revealed indifference relation as: $x \sim^* y \Leftrightarrow$ there is some $B \in \mathcal{B}$ such that $x,y \in B$, $x \in \mathrm{C}(B)$ and $y \in \mathrm{C}(B)$. Further suppose that the weak axiom holds.
a) Prove that $x \sim^* y$.
b) If $\succsim$ rationalises $C(\cdot)$ relative to $\mathcal{B}$, then prove that $x \sim y$.

9. Mas-Colell, Whinston and Green: Exercises 1.B.3, 1.C.1, 1.C.2, 1.C.3, 1.D.3