

Problem Set 3
Classical Demand Theory

1. Assume the indirect utility function $v(p_1, p_2, w) = w/(p_1 + p_2)$.
 - a) Is $v(\cdot)$ homogeneous of degree zero? Is it quasiconvex?
 - b) Use Roy's Identity to derive the Walrasian demand function and compute the Slutsky substitution matrix.
2. Assume the indirect utility function $v(p_1, p_2, w) = w/2p_1^{1/2}p_2^{1/2}$.
 - a) Find the Walrasian demand function and check if it satisfies Walras' Law.
 - b) Would it be allowed to take some strictly increasing transformation of $v(\cdot)$, if one found that this would simplify the calculations in (b)? Justify your answer.
3. In an economy with 2 goods only, assume a consumer with utility function $u(\mathbf{x}) = x_1x_2$. The prices of goods 1 and 2 are $p_1 > 0$ and $p_2 > 0$ respectively. The required level of utility for this consumer is $u = 1$.
 - a) Find the Hicksian demand function.
 - b) Calculate the Slutsky matrix using the Hicksian demand function you found in (a).
 - c) Write down and solve the dual utility maximisation problem.
4. In an economy with $L = 2$, the expenditure function of an individual is $e(\mathbf{p}, u) = 2u^{1/2}p_1^{1/2}p_2^{1/2}$.
 - a) Find the Hicksian demand function and the indirect utility function.
 - b) Calculate the Slutsky matrix.
 - c) If $p_1 = 4$, $p_2 = 1$ and $w = 4$, use the duality properties to find the level of Walrasian demand for good 2.
 - d) Find the Walrasian demand function and verify your answer in (c).
5. a) Figure 3.G.2 of Mas-Colell, Whinston and Green (page 73) shows a scenario where the price of commodity 1 changes from p to p' , $p' > p$ and demonstrates that the Slutsky wealth compensation $\Delta w_{\text{Slutsky}}$ is greater than the Hicksian wealth compensation Δw_{Hicks} . Demonstrate the same for the case $p' < p$. Then, prove that $\Delta w_{\text{Hicks}} \leq \Delta w_{\text{Slutsky}}$ for any price change from \mathbf{p} to \mathbf{p}' .
6. Do choices in example 2.F.1 of M/W/G (p.35) violate the strong axiom of revealed preference?
7. For each one of the following statements, argue whether they are true, false or uncertain. Justify your answers.
 - a) In an economy with only two commodities, if the consumers' choices are consistent with the weak axiom of revealed preference, then we can always find a rationalising preference relation.
 - b) The Walrasian demand functions we derive from Roy's identity are contingent on the chosen representation for the indirect utility function.
 - c) In equilibrium, the marginal rate of substitution (MRS_{12}) is always equal to p_1/p_2 .
 - d) The solution(s) of the utility maximisation problem must satisfy Walras' Law, and hence, the constraint $\mathbf{p} \cdot \mathbf{x} \leq w$ might as well be written as $\mathbf{p} \cdot \mathbf{x} = w$.
 - e) There can be no utility function to represent a preference relation that does not satisfy local nonsatiation.
8. Mas-Colell, Whinston and Green: Exercises 3.B.2, 3.C.1, 3.C.6, 3.D.5, 3.D.6, 3.E.4, 3.E.6, 3.G.1, 3.G.3, 3.G.6, 3.G.14, 3.G.15, 3.J.1